Math 10/11 Honors: Section 3.3 Finite and Infinite Geometric Series

1. Find the sum for the following geometric series. Show all your work and steps:

a)
$$S = 8 + 4 + 2 + \dots \frac{1}{128}$$

b)
$$S = 3 + 6 + 12 + \dots 3072$$

c)
$$S = \sqrt{2} + 2 + 2\sqrt{2} + \dots 256$$

d)
$$S = -\frac{1}{8} + 0.25 + -0.5 + \dots 256$$

e)
$$S = \frac{64}{81} + \frac{32}{27} + \frac{16}{9} + \dots \frac{729}{16}$$

f)
$$S = \frac{125}{64} - \frac{25}{16} + \frac{5}{4} - \dots + \frac{1024}{3125}$$

2. Given that each of the following series are infinite, determine the sum. Show all your work and steps.

a)
$$S = 27 + 9 + 3 + 1 + \dots$$

b)
$$S = 2 + 1.8 + 1.62 + 1.458 + \dots$$

c)
$$S = 0.4 + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \dots$$

d)
$$S = 11 + 16 + 8 + 4 + 2 + \dots$$

e)
$$S = 256 - 128\sqrt{2} + 128 - 2\sqrt{64} + \dots$$

f)
$$S = \frac{64}{125} + \frac{16}{25} + \frac{4}{5} + \dots$$

3. Given the information of a geometric sequence, find the indicated unknown value:

a)
$$S_2 = 5$$
, $S_4 = 85$, $r =$

b)
$$S_3 = 19$$
, $S_{\infty} = 27$, $r =$

c)
$$a = 12$$
, $r = 2$, $S_n = 762$, $n = ?$

d)
$$t_2 = 3x$$
, $t_3 = 2x - 1$, $t_4 = 7x + 8$, $S_6 =$

e)
$$a = x + 2$$
, $t_2 = 3x$, $t_3 = x^2 + 8$, $S_5 =$

f)
$$1+x+x^2+x^3+....=20$$
; $x=?$

4. What is the sum of the following geometric series?

$$12 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \dots$$

5. The sum of the first 8 terms of a geometric series is 1020 with a common ratio of -2. Determine the first term.

6. The sum of an infinite geometric series is 1 and the common ratio is $-\frac{2}{5}$, determine the 3rd term.

7. Determine the 8th term of an infinite geometric series with $S_{\infty}=24$ and $r=\frac{3}{4}$

8. A ball is dropped from a height of 2.0m to a floor. After each bounce, the ball rises to 63% of its previous height. What is the total vertical distance the ball has travelled after 5 bounces? What is the total vertical distance the ball travelled after it comes to rest?

9. If the sum of a geometric series is given by the formula $S_n = 4 - 8(-7)^{n-1}$, determine the value of t_5 .

10. An oil well produces 30,000 barrels of oil during its first month of production. Suppose its production drops by 5% each month. Estimate the total production before the well runs dry.

11. If the sum of "n" terms of a geometric series is $S_n = 2(3^n - 1)$, determine the 5th term of this series.

12. A contest winner is given two prizes to choose from. Prize A is given \$20,000,000 right away. Prize "B" is given \$1 in the first year, \$2 in the next year, \$4, \$8, each following year for the next 30 years. After how many years will sum of Prize B surpass the Prize A?

13. For any geometric series, what is the value of $S_{n+1}-S_n$ equal to?

14. The sum of the 1st and 2nd term of a geometic sequence is 4 and the sum of the 3rd and 4th term is 36. Determine the sum of the first 8 terms.

15. Let $S_n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{n-1}}$. Prove algebraically that S_n is less than "2" for all values of "n". Justify your answer.

16. Let $S_n = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{3^{n-1}}$. Prove algebraically that S_n is less than "1.5" for all values of "n". Justify your answer.

- 17. For what values of "x" will the series have a finite sum? $1 + \left(\frac{x-2}{3}\right) + \left(\frac{x-2}{3}\right)^2 + \left(\frac{x-2}{3}\right)^3 + \dots$
- 18. The geometric series $a + ar + ar^2 + ...$ has a sum of 7, and the terms involving odd powers of "r" have a sum of 3. What is a + r?
- 19. In a sequence of numbers, the sum of the first "n" terms is equal to $5n^2 + 6n$. What is the sum of the 3rd, 4th, and 5th terms of the original sequence.

20. Challenge: AIME II 2012 #2 Two geometric sequences a_1, a_2, a_3, \ldots and b_1, b_2, b_3, \ldots have the same common ratio, with $a_1 = 27$, $b_1 = 99$, and $a_{15} = b_{11}$. Find the value of a_9

21. Given an geometric series where $\frac{S_4}{S_6} = \frac{117}{133}$ and all six terms are integers, what is the first term and the common ratio?